Name ______________________________ Period __________ Date _____________

6-7
STUDENT PACKET

MATHLINKS: GRADE 6
STUDENT PACKET 7
FRACTION MULTIPLICATION AND DIVISION

WORD BANK

Word or Phrase	Definition or Description	Example or Diagram
commutative property of multiplication		
distributive property		
divisor		
factor		
multiplicative identity property		
product		
quotient		
reciprocal		
remainder		

FRACTION MULTIPLICATION

Summary

We will use whole number multiplication concepts to provide meaning for fraction multiplication. We will develop procedures to multiply fractions.

Goals

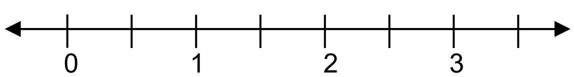
- Extend concepts of whole number multiplication to fraction multiplication.
- Use an area model and linear model to explain fraction multiplication concepts.
- Develop fraction multiplication procedures.

Warmup

Write each multiplication expression as a repeated addition expression. Then find the sum.

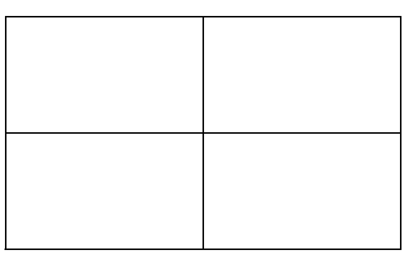
1. 6(5) = ____ + ____ + ____ + ____ + ____ + ____ = ______

2. Show that $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2\frac{1}{2}$ using a number line.



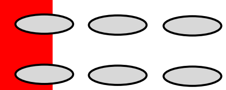
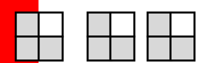
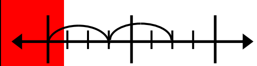
3. Compute: $1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4}$

4. Compute 38(72) using an area model (rectangle not drawn to scale).



FRACTION MULTIPLICATION: GROUPING

Multiplication can be interpreted as forming equal groups. Complete the table.

	Multiplication Expression	Interpretation	Diagram	Addition Expression	Product
1.	$2 \bullet 3$	2 groups of 3		____ + ____	
2.	$3 \bullet \frac{3}{4}$	3 groups of $\boxed{\frac{\ }{\ }}$			
3.		___ groups of $\boxed{\frac{\ }{\ }}$			
4.	$2 \bullet \frac{1}{5}$				
5.		4 groups of $\frac{2}{3}$			
6.	$6 \bullet \frac{2}{3}$				
7.				$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$	
8.				$\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$	

9. Create a story problem for $4 \bullet \frac{1}{2}$ and use a diagram to show the product.

COMMUTATIVE PROPERTY OF MULTIPLICATION

Explore the meaning of products where the order of the multiplication is reversed.

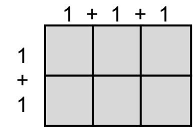
	Multiplication Expression	Interpretation	Diagram	Addition Expression	Product
1.	$2 \bullet 4$	2 groups of 4			
	$4 \bullet 2$	4 groups of 2			
2.	$3 \bullet \frac{1}{6}$				
	$\frac{1}{6} \bullet 3$				
3.		$\frac{2}{3}$ group of 9			
		9 groups of $\frac{2}{3}$			

4. Does $a \bullet b$ give the same result as $b \bullet a$? _____ Use problem 2 above to explain.

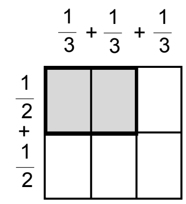
5. Does $a \bullet b$ mean the same thing as $b \bullet a$? _____ Use problem 3 above to explain.

FRACTION MULTIPLICATION: AREA MODEL

Multiplication can also be explained with an area model.
The 2 by 3 rectangle at the right has an area of $2 \times 3 = 6$

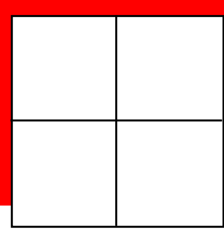


An area model is also useful for multiplying proper fractions. The square at the right has side lengths of 1 unit. A rectangle that is $\frac{1}{2}$ by $\frac{2}{3}$ is shaded. The shaded area shows that $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$.



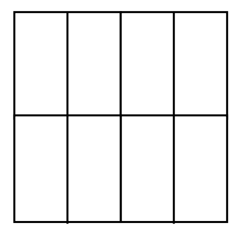
Use an area model to find each product. Let each large square represent a 1 unit by 1 unit square. Do not simplify products.

1. Mark the side lengths and shade a rectangle that is $\frac{1}{2}$ by $\frac{1}{2}$.



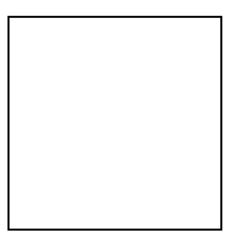
$\frac{1}{2} \times \frac{1}{2} = \boxed{\frac{\quad}{\quad}}$

2. Mark the side lengths and shade a rectangle that is $\frac{1}{2}$ by $\frac{1}{4}$.



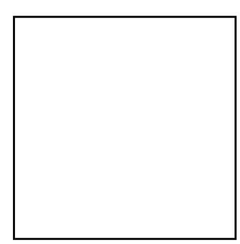
$\frac{1}{2} \times \frac{1}{4} = \boxed{\frac{\quad}{\quad}}$

3. Mark the side lengths and shade a rectangle that is $\frac{1}{4}$ by $\frac{3}{4}$.



$\frac{1}{4} \times \frac{3}{4} = \boxed{\frac{\quad}{\quad}}$

4. Mark the side lengths and shade a rectangle that is $\frac{2}{3}$ by $\frac{3}{4}$.



$\frac{2}{3} \times \frac{3}{4} = \boxed{\frac{\quad}{\quad}}$

FRACTION MULTIPLICATION: AREA MODEL (Continued)

Use an area model to find each product.

5. $\frac{1}{2} \times \frac{1}{6}$	6. $\frac{1}{2} \times \frac{2}{3}$	7. $\frac{2}{5} \times \frac{2}{3}$

8. Look closely at the relationship between the factors and the products in problems 5-7 above. Complete each sentence.

   The numerators of the factors are ______________________ to arrive at the numerator of the product.

   The denominators of the factors are ______________________ to arrive at the denominator of the product.

9. From what you have discovered, state a fraction multiplication rule.

   $\frac{a}{b} \times \frac{c}{d} = \boxed{\frac{\quad}{\quad}}$ This is called the multiply-across rule.

Use the multiply-across rule to find each product.

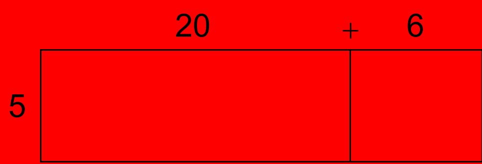
10. $\frac{1}{4} \times \frac{1}{4}$	11. $\frac{1}{3} \times \frac{1}{4}$	12. $\frac{1}{2} \times \frac{2}{5}$

13. Create a story problem for $\frac{4}{5} \times \frac{2}{3}$ and use a visual fraction model to show the product.

THE DISTRIBUTIVE PROPERTY

Use an area model to find each product (rectangles are not drawn to scale.)

1. $5 \times 26 = 5 \times (20 + 6)$

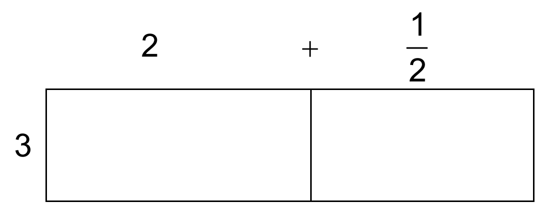


$5 \times (20 + 6)$ = ___•___ + ___•___

= ______ + ______

= ______

2. $3 \times 2\frac{1}{2} = (3)\left(2 + \frac{1}{2}\right)$



$3\left(2 + \frac{1}{2}\right)$ = ___•___ + ___•___

= ______ + ______

= ______

3. Describe how the distributive property is used in the two problems above.

Use an area model (not to scale) and the distributive property to multiply.

4. $7 \times 2\frac{1}{2}$	5. $1\frac{1}{8} \times 6$	6. $12 \times 3\frac{2}{5}$

7. Jamil multiplied $3 \bullet 2\frac{1}{2}$ this way.

Is Jamil's answer correct? ______

Did Jamil use the distributive property to get the result? ______ Explain.

$$(3)(2\tfrac{1}{2}) = 2\tfrac{1}{2} + 2\tfrac{1}{2} + 2\tfrac{1}{2}$$
$$= 2v + \frac{1}{2} + 2 + \frac{1}{2} + 2 + \frac{1}{2}$$
$$= 6 + \frac{3}{2} = 7\frac{1}{2}$$

MULTIPLYING MIXED NUMBERS

Here are three diagrams (not to scale) that illustrate the product $2\frac{1}{2} \times 2\frac{1}{2}$.

Diagram 1

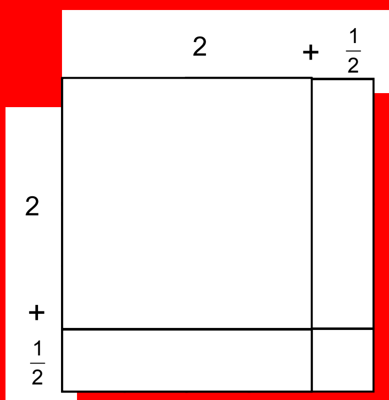


Diagram 2

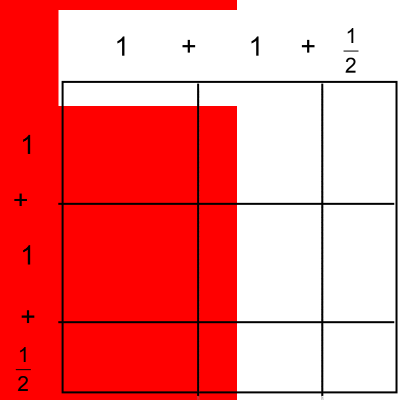
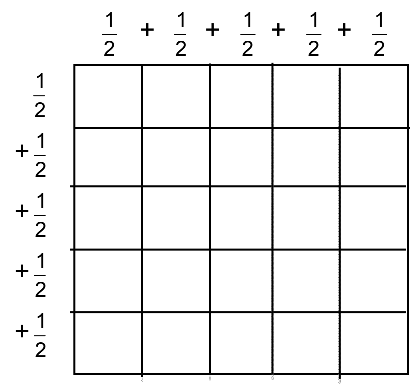


Diagram 3



1. Write the area of each small rectangle inside each diagram.

2. For Diagram 1, explain how you found the area of the lower right corner rectangle.

3. Compute $2\frac{1}{2} \times 2\frac{1}{2}$ using Diagram 1.

4. Compute $2\frac{1}{2} \times 2\frac{1}{2}$ using Diagram 2.

5. Compute $2\frac{1}{2} \times 2\frac{1}{2}$ using Diagram 3.

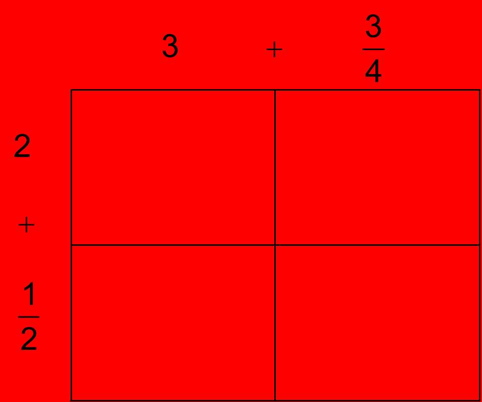
6. Which diagram best illustrates the multiply-across rule? _______ Explain.

7. Which diagram best illustrates the distributive property? _____ Explain.

MULTIPLYING MIXED NUMBERS (Continued)

8. Multiply $2\frac{1}{2} \times 3\frac{3}{4}$ using the two methods below.

a. Find the area of each smaller rectangle and add the areas together.



Rectangle is not drawn to scale.

$2\frac{1}{2} \times 3\frac{3}{4} =$ ______ + ______ + ______ + ______

= ____________________

b. Change each mixed number to an improper fraction. Then use the multiply-across rule.

$2\frac{1}{2} =$ ______ $3\frac{3}{4} =$ ______

$2\frac{1}{2} \times 3\frac{3}{4} =$

$\frac{}{} \times \frac{}{} = \frac{}{}$

9. Multiply using an area model.

a. $2\frac{2}{3} \times 1\frac{1}{4}$

Check using the multiply-across rule:

b. $4 \times 1\frac{1}{2}$

Check using repeated addition:

FRACTION MULTIPLICATION PRACTICE 1

Compute using repeated addition, an area model, or the multiply-across rule.

1. $4 \bullet \frac{5}{8}$	2. $\frac{3}{5} \times \frac{5}{12}$	3. $8\left(2\frac{1}{3}\right)$
4. $1\frac{1}{3} \bullet \frac{2}{9}$	5. $1\frac{1}{4} \times 1\frac{1}{15}$	6. $\frac{4}{9}\left(\frac{3}{16}\right)$

7. Find the area of a rug with dimensions $5\frac{1}{4}$ yards by $4\frac{1}{3}$ yards.

8. Raoul intended to run $3\frac{3}{4}$ miles, but he only completed $\frac{2}{3}$ of the distance. How far did he run?

MULTIPLICATION AND THE "BIG 1"

Use the multiply-across rule with the "big 1"	Use the "big 1" shortcut notation
$\frac{2}{3} \bullet \frac{3}{4} = \frac{2 \bullet 3}{3 \bullet 2 \bullet 2}$ $= \frac{2 \bullet 3}{2 \bullet 2 \bullet 3}$ $= \frac{1 \bullet 1}{2 \bullet 1 \bullet 1}$ $= \frac{1}{2}$	$\frac{2}{3} \bullet \frac{3}{4} = \frac{{}^{1}\cancel{2}}{{}_{1}\cancel{3}} \bullet \frac{\cancel{3}^{1}}{\cancel{4}_{2}}$ $= \frac{1 \bullet 1}{1 \bullet 2}$ $= \frac{1}{2}$

Compute using an appropriate strategy.

1. $\frac{3}{5} \times \frac{5}{6}$	2. $\frac{3}{10} \bullet \frac{5}{9}$	3. $4\left(\frac{3}{7}\right)$
4. $6\left(\frac{5}{8}\right)$	5. $6 \bullet 4\frac{2}{3}$	6. $2\frac{2}{3} \times 1\frac{1}{2}$

FRACTION MULTIPLICATION PRACTICE 2

Compute.

Use repeated addition	Use the multiply-across rule	Use the multiply-across rule with "the big 1"
1. $5 \bullet \frac{1}{4}$	2. $\frac{4}{5} \times \frac{1}{3}$	3. $\left(\frac{3}{8}\right)\left(\frac{4}{5}\right)$

Compute using an appropriate strategy.

4. $\frac{3}{8} \times \frac{4}{9}$	5. $3\left(\frac{5}{6}\right)$	6. $\frac{2}{15} \bullet \frac{5}{8}$

7. One smoothie uses $2\frac{1}{2}$ cups of bananas. How many cups of bananas are needed for 8 smoothies?

8. Find the area of square bathmat with a side length of $\frac{4}{5}$ of a yard.

FRACTION DIVISION 1

Summary

We will explore division of fractions in measuring contexts, and explore whether the multiply-across rule for multiplication can be extended to division.

Goals

- Explore the meaning of fraction division.
- Understand and use the divide-across rule for fraction division.
- Observe that division and multiplication are inverse operations.
- Solve problems involving division of fractions.
- Interpret the meaning of the remainder in a fraction division situation.

Warmup

1. Manny has 8 cups of strawberries. He wants to put 2 cups of strawberries into each fruit smoothie that he makes. How many fruit smoothies can Manny make?

   a. Draw a diagram to illustrate the problem.

   b. Fill in numbers to rephrase the problem as a measure-out division problem.

   How many (groups of) ____ are in ____?

   c. Write a division statement that illustrates this relationship.

2. Minnie has 8 cups of strawberries. She wants to put $\frac{1}{2}$ cup of strawberries into each fruit smoothie that she makes. How many fruit smoothies can Minnie make?

   a. Draw a diagram to illustrate the problem.

   b. Fill in numbers to rephrase the problem as a measure-out division problem.

   How many (groups of) ____ are in ____?

   c. Write a division statement that illustrates this relationship.

FRUIT SMOOTHIE PROBLEMS

1. Monica has $1\frac{1}{3}$ cups of blueberries. She wants to put $\frac{2}{3}$ of a cup of blueberries into each fruit smoothie that she makes. How many fruit smoothies can Monica make?

   a. Draw a diagram to represent the problem.

   b. How many (full) fruit smoothies can Monica make?

   c. How many cups of blueberries will be left over?

   d. What fraction of a fruit smoothie can be made with the leftover blueberries?

   e. Write a division statement for the fruit smoothie problem.

   f. Recall that multiplication and division are inverse operations. Check that your division statement is true by rewriting it as a multiplication statement.

2. Max has $1\frac{2}{3}$ cups of blueberries. He wants to put $\frac{2}{3}$ of a cup of blueberries into each fruit smoothie that he makes. How many fruit smoothies can Max make?

   a. Draw a diagram to represent the problem.

   b. How many (full) fruit smoothies can Max make?

   c. How many cups of blueberries will be left over?

   d. What fraction of a fruit smoothie can be made with the leftover blueberries?

   e. Write a division statement for the fruit smoothie problem.

   f. Check that your division statement is true by rewriting it as a multiplication statement.

A FRACTION DIVISION RULE

In fraction multiplication, we multiply-across to get the product of two factors. Do you think we can divide-across in a division problem to get the quotient of two numbers? Let's try it with the smoothie problems.

1. Monica has $1\frac{1}{3}$ cups of blueberries. She wants to put $\frac{2}{3}$ cup of blueberries into each fruit smoothie that she makes. How many fruit smoothies can Monica make?

$$1\frac{1}{3} \div \frac{2}{3} = \frac{\square}{3} \div \frac{2}{3}$$

$$= \frac{\square \div 2}{3 \div \square}$$

$$= \frac{\square \div 2}{\square}$$

$$= \square \div 2$$

$$= \square$$

a. Did the calculation agree with your diagram on the previous page?

b. How can you tell from the divide-across calculation the number of (full) fruit smoothies that can be made?

c. How can you tell from the calculation that there will be no extra fruit smoothie?

2. Max has $1\frac{2}{3}$ cups of blueberries. He wants to put $\frac{2}{3}$ cup of blueberries into each fruit smoothie that he makes. How many fruit smoothies can Max make?

$$1\frac{2}{3} \div \frac{2}{3} = \frac{\square}{3} \div \frac{\square}{3}$$

$$= \frac{\square \div \square}{\square \div \square}$$

$$= \frac{\square \div \square}{1}$$

$$= 5 \div 2$$

$$= \square$$

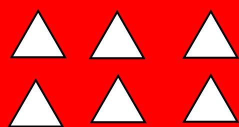
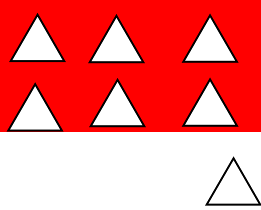
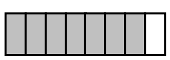
a. Did the calculation agree with your diagram on the previous page?

b. How can you tell from the divide-across calculation the number of (full) fruit smoothies that can be made?

c. What does $\frac{1}{2}$ represent in the quotient?

DIVIDE-ACROSS

Complete the chart below.

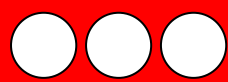
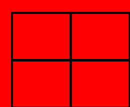
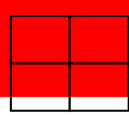
Words	Diagram	Division statement	Compute using a divide-across strategy
1. How many groups of 3 are in 6?			______ ÷ ______ = ______
2. How many groups of $\frac{3}{8}$ are in $\frac{6}{8}$?		$\frac{6}{8} \div \frac{3}{8} = \square$	$\frac{6}{8} \div \frac{3}{8} = \frac{6 \div 3}{8 \div 8} = \_\_\_\_ = \square$
3. How many groups of 3 are in 7?			
4. How many groups of $\frac{3}{8}$ are in $\frac{7}{8}$?			

5. For problem 2 above, what is the result in the denominator when dividing across?

6. Problems 3 and 4 above have a fractional remainder of $\frac{1}{3}$. To what does the $\frac{1}{3}$ refer?

DIVIDE-ACROSS (Continued)

Sometimes to divide-across it is helpful to find a common denominator first.

Words	Diagram	Division statement	Compute using a divide-across strategy
7. How many (groups of) $\frac{1}{2}$ are in 3?		$3 \div \frac{1}{2} = \square$	(Find a common denominator first.) $\frac{6}{2} \div \frac{1}{2} = \boxed{\frac{\quad}{\quad}} =$
8. How many groups of $\frac{1}{2}$ are in $\frac{3}{4}$?			
9. How many groups of $\frac{3}{4}$ are in $\frac{1}{2}$?			

10. These examples illustrate the divide-across rule for dividing fractions. When fractions have a common denominator:

$$\frac{a}{b} \div \frac{c}{b} = \frac{a \div c}{b \div b} = \frac{a \div c}{\square} = \boxed{\quad\quad} \text{ or } \frac{\square}{\square} \; (b \neq 0, c \neq 0).$$

Use the divide-across rule to solve each division problem.

11. $4 \div \frac{1}{2}$ (Hint: $4 = \frac{8}{2}$)	12. $4\frac{1}{2} \div 1\frac{1}{8}$ (Hint: use improper fractions)	13. $2\frac{1}{8} \div \frac{3}{4}$

FRACTION DIVISION PRACTICE

1. A 2-foot-long sandwich is cut into portions that are $\frac{3}{4}$ feet long each. Find out how many portions can be cut and what fraction of a portion is leftover.

   a. Write a division problem.

   b. Solve using a diagram.

   c. Solve using a divide-across strategy.

   d. How many portions can be cut?

   e. How long is the piece that is leftover?

   f. What fraction of a portion is leftover?

   g. Check your solution by multiplication.

2. A 4-foot-long board is cut into shelves that are $1\frac{1}{4}$ feet long each. Find out how many shelves can be cut and what fraction of a shelf is leftover.

   a. Write a division problem.

   b. Solve using a diagram.

   c. Solve using a divide-across strategy.

   d. How many shelves can be cut?

   e. How long is the piece that is leftover?

   f. What fraction of a shelf is leftover?

   g. Check your solution by multiplication.

3. Divide $2\frac{5}{8}$ by $3\frac{1}{2}$. Check your solution using multiplication.

FRACTION DIVISION 2

Summary	Goals
We will explore division of fractions in measuring and fair-share contexts, and learn an efficient rule for dividing fractions.	• Explore the meaning of fraction division. • Understand and use the multiply-by-the-reciprocal rule for fraction division. • Solve problems involving division of fractions. • Interpret the meaning of the remainder in a fraction division situation.

Warmup

Use the divide-across rule to compute.

1. $\frac{4}{9} \div \frac{1}{3}$

2. $3 \div 1\frac{1}{5}$

3. Minnie has 8 cups of strawberries. She wants to divide them equally to make 2 fruit smoothies. How many cups of strawberries will Minnie put in each smoothie?

   a. Draw a diagram to illustrate the problem.

   b. Write a division statement that illustrates this relationship.

   c. Mickey says that he can solve Minnie's problem by computing $8 \bullet \frac{1}{2}$. Is he correct? Explain.

GRANOLA BAR PROBLEMS

1. Three girls want to share one granola bar so that each person gets the same amount. How much will each girl get?

   a. Draw a diagram to illustrate the problem.

   b. Fill in numbers to rephrase the problem as a fair-share division problem. How can the girls divide

   _____ into ______ equal groups?

   c. Write a division statement that illustrates this relationship.

   d. How much of a granola bar does each girl get?

   e. Multiply: $1 \bullet \frac{1}{3}$

   f. The results from (c) and (e) suggest that:

   Dividing by 3 gives the same result

   as multiplying by ______.

2. Four girls want to share $\frac{1}{2}$ of a granola bar so that each one gets the same amount. How much will each girl get?

   a. Draw a diagram to illustrate the problem.

   b. Fill in numbers to rephrase the problem as a fair-share division problem. How can the girls divide

   _____ into ______ equal groups?

   c. Write a division statement that illustrates this relationship.

   d. How much of a granola bar does each girl get?

   e. Multiply: $\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)$

   f. The results from (c) and (e) suggest that:

   Dividing by ______ gives the same

   result as multiplying by ______.

HOME IMPROVEMENT PROBLEMS

1. Sirena has ribbon that is one foot long. She wants to cut pieces that are each $\frac{1}{4}$ foot long for an art project. How many pieces can she cut?

   a. Draw a diagram to illustrate the problem.

   c. Fill in numbers to rephrase the problem as a fair-share division problem. How can Sirena divide

   ____ into _____ equal groups?

   c. Write a division problem that illustrates this relationship.

   d. How many pieces can she cut?

   e. Multiply: $1 \times \frac{4}{1}$

   f. The results from (c) and (e) suggest that:

   Dividing by _____ gives the same

   result as multiplying by _____.

2. Olivia has $\frac{2}{3}$ yard of fabric to make pillows. If each pillow requires $\frac{1}{6}$ yard of fabric, how many pillows can she make?

   a. Draw a diagram to illustrate the problem.

   d. Fill in numbers to rephrase the problem as a fair-share division problem. How can Olivia divide

   ____ into _____ equal groups?

   b. Write a division problem that illustrates this relationship.

   d. How many pillows can she make?

   e. Multiply: $\frac{2}{3} \times \frac{6}{1}$

   f. The results from (c) and (e) suggest that:

   Dividing by _____ gives the same

   result as multiplying by _____.

GRANOLA BAR PROBLEM PRACTICE

1. Three girls want to share two granola bars so that each person gets the same amount. How much will each girl get?

   a. Draw a diagram to illustrate the problem.

   e. Fill in numbers to rephrase the problem as a fair-share division problem. How can the girls divide

   _____ into ______ equal groups?

   b. Write a division problem that illustrates this relationship.

   c. How much of a granola bar does each girl get?

   d. Multiply: $2\left(\frac{1}{3}\right)$

2. Two girls want to share $1\frac{1}{2}$ granola bars so that each one gets the same amount. How much will each girl get?

   a. Draw a diagram to illustrate the problem.

   f. Fill in numbers to rephrase the problem as a fair-share division problem. How can the girls divide

   _____ into ______ equal groups?

   b. Write a division problem that illustrates this relationship.

   c. How much of a granola bar does each girl get?

   d. Multiply: $1\frac{1}{2} \bullet \frac{1}{2}$

3. Explain how parts (c) and (e) are related for both problems above.

HOME IMPROVEMENT PROBLEM PRACTICE

1. Alexander has a board that is three yards long. He wants to cut pieces that are each $\frac{3}{4}$ yard to make shelves. How many pieces can he cut?

   a. Draw a diagram to illustrate the problem.

   g. Fill in numbers to rephrase the problem as a fair-share division problem. How can Alexander divide

   ____ into _____ equal groups?

   b. Write a division problem that illustrates this relationship.

   c. How many pieces can he cut?

   d. Multiply: $3 \times \frac{4}{3}$

2. Michael has $4\frac{2}{3}$ yards of wire. He cuts it into pieces that are each $\frac{1}{3}$ yard and uses each piece to tie up a plant in the garden. How many pieces can he cut?

   a. Draw a diagram to illustrate the problem.

   h. Fill in numbers to rephrase the problem as a fair-share division problem. How can Michael divide

   ____ into _____ equal groups?

   b. Write a division problem that illustrates this relationship.

   c. How many pieces can he cut?

   d. Multiply: $4\frac{2}{3} \times \frac{3}{1}$

3. Generalize the results from parts (c) and (e) above.

FRACTION DIVISION RULE

In the Granola problems and Home Improvement problems on pages 19-22, you observed that division by a number gives the same result as multiplying by its reciprocal. This is because division and multiplication are inverse operations. The reciprocal of a is $\frac{1}{a}$.

Write the reciprocal of each number:

1. 4	2. $\frac{1}{5}$	3. $\frac{2}{3}$	4. $3\frac{1}{10}$

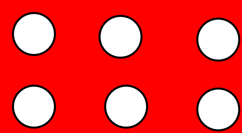
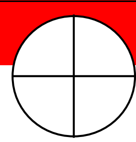
Recall the Fruit Smoothie problems on pages 13-14 from the previous lesson. Complete each division calculation by multiplying by the reciprocal of the divisor. Then answer the questions. Your calculations should agree with those you made before.

5. Monica has $1\frac{1}{3}$ cups of blueberries. She wants to put $\frac{2}{3}$ cup of blueberries into each fruit smoothie that she makes. How many fruit smoothies can Monica make?	$1\frac{1}{3} \div \frac{2}{3} = \frac{\boxed{}}{3} \div \frac{2}{3}$ $= \frac{\boxed{}}{3} \times \frac{3}{\boxed{}}$ $=$ ______
6. Max has $1\frac{2}{3}$ cups of blueberries. He wants to put $\frac{2}{3}$ cup of blueberries into each fruit smoothie that he makes. How many fruit smoothies can Max make?	$1\frac{2}{3} \div \frac{2}{3} = \frac{\boxed{}}{3} \div \frac{\boxed{}}{3}$ $= \boxed{\frac{}{}} \times \boxed{\frac{}{}}$ $=$

We will call this the multiply-by-the-reciprocal rule.

MULTIPLY-BY-THE-RECIPROCAL

Complete the chart below.

Words	Diagram	Division equation	Multiplication equation
1. How many groups of 3 are in 6?		$6 \div \square = 2$	$6 \times \square = 2$
2. How many groups of 2 are in 3?		$3 \div 2 = \square$	$3 \times \square = 1\frac{1}{2}$
3. How many groups of 3 are in 2?			
4. How many groups of $\frac{1}{4}$ are in $\frac{3}{4}$?		$\frac{3}{4} \div \square = \square$	
5. How many groups of $\frac{3}{4}$ are in $\frac{1}{4}$?		$\frac{1}{4} \div \frac{3}{4} = \square$	

6. The examples above illustrate a rule for division:

Dividing by a number gives the same result as multiplying by its ________________.

For division of fractions, this rule can be written:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \boxed{\frac{\quad}{\quad}} \quad (b \neq 0,\ c \neq 0,\ d \neq 0).$$

We call this the multiply-by-the-reciprocal rule.

MULTIPLY-BY-THE-RECIPROCAL (Continued)

Use the multiply-by-the-reciprocal rule to find each quotient.

7. $4 \div \frac{1}{2}$	8. $2\frac{1}{8} \div \frac{3}{4}$	9. $4\frac{1}{2} \div 1\frac{1}{8}$

10. Hector runs 3 miles around the perimeter of the school. One lap around the school is $\frac{2}{3}$ miles. How many full laps around the school does he run? What fraction of a lap is left over?

a. Solve using a diagram.	b. Solve using the multiply-by-the-reciprocal rule.

11. Create a story problem for $\frac{2}{3} \div 4$, and use a visual fraction model to show the quotient.

FRACTION DIVISION PRACTICE

Compute using the divide-across rule.

1. $\frac{3}{4} \div \frac{9}{4}$	2. $3\frac{2}{3} \div \frac{1}{6}$	3. $\frac{1}{2} \div \frac{1}{3}$

Compute using the multiply-by-the-reciprocal rule.

4. $1\frac{2}{5} \div \frac{3}{5}$	5. $1\frac{7}{21} \div \frac{4}{11}$	6. $3\frac{1}{3} \div \frac{4}{9}$

Compute using a rule of your choice.

7. $4 \div 1\frac{1}{3}$	8. $1\frac{2}{8} \div 1\frac{2}{16}$	9. $2\frac{2}{15} \div \frac{4}{5}$

10. In problem 8, which rule did you choose to use? ________________ Why?

11. Create a story problem for $6 \div \frac{3}{4}$ and use a visual fraction model to show the quotient.

SKILL BUILDERS, VOCABULARY, AND REVIEW

SKILL BUILDER 1

Use an area model to find the products below.

1. $53(72) =$ _____

2. $205 \bullet 34 =$ _____

Use <, >, or = symbols to make each statement true.

3. 2^3 ___ 3^2

4. 4^2 ___ 2^4

5. 5^3 ___ 3^4

Use replicating or splitting diagrams to illustrate that the following equations are true.

6. $\frac{2}{3} = \frac{6}{9}$

7. $\frac{3}{4} = \frac{15}{20}$

8. Heidi wants to give out free samples of her freshly made kale and ginger juice. She has 1,250 ml of juice to give away.

   If she wants to give away 80 ml samples, how many samples can she make?

   Is there any juice left over? If so, how much?

SKILL BUILDER 2

1. Arrange the rational numbers below in order from least to greatest.

   $\frac{2}{3}, \frac{2}{2}, \frac{5}{9}, \frac{5}{6}, \frac{1}{2}$

2. Write four fractions that are equivalent to $\frac{10}{20}$.

Calculate each sum.

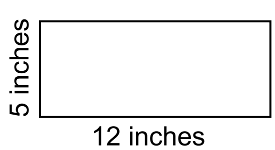
3. $\frac{3}{4} + 1\frac{4}{5}$

4. $3 + 1\frac{1}{3} + 2\frac{1}{2}$

5. Reseda claims that $\frac{13}{4}$ is greater than $\frac{10}{3}$ because 13 and 4 are greater than 10 and 3. Show her that $\frac{10}{3}$ is greater than $\frac{13}{4}$ and offer an explanation that would help her understand her error.

6. Which quantity is greater, $\frac{1}{2}$ or $3 - \frac{2}{3}$? Explain your reasoning by using calculations or diagrams.

7. Find the perimeter and area of the rectangle below. Make sure you use appropriate units.



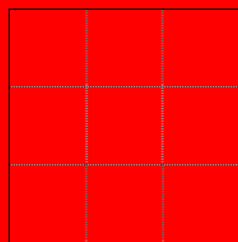
Perimeter: ____________________

Area: ________________________

SKILL BUILDER 3

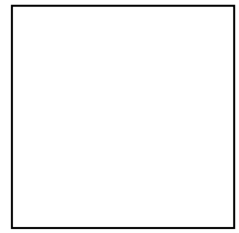
Use an area model to find each product in problems 1 and 2. Let each original square figure represent a 1 unit by 1 unit square. Do not simplify products.

1. Mark the side length and shade a rectangle that is $\frac{1}{3}$ by $\frac{1}{3}$.



$\frac{1}{3} \times \frac{1}{3} =$

2. Mark the side lengths and shade a rectangle that is $\frac{4}{5}$ by $\frac{1}{2}$.



$\frac{4}{5} \times \frac{1}{2} =$

For problems 3-5, use the multiply-across rule to find each product.

3. $\frac{3}{5} \times \frac{2}{5}$	4. $\frac{3}{4} \bullet \frac{1}{2}$	5. $\left(\frac{1}{2}\right)\left(\frac{5}{6}\right)$

6. Ricardo thinks that $\frac{1}{3}$ of $\frac{1}{2}$ is equivalent to $\frac{1}{2}$ of $\frac{1}{3}$. Use diagrams or calculations to prove Ricardo's conjecture.

Find the differences.

7. $1\frac{4}{5} - \frac{3}{4}$

8. $5\frac{1}{3} - 2\frac{1}{2}$

SKILL BUILDER 4

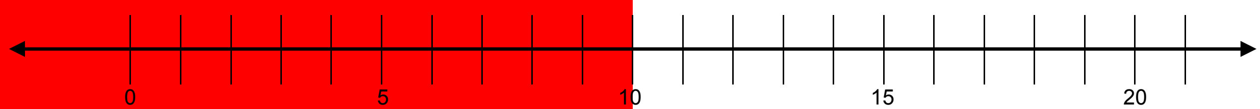
Coach Watson wanted his basketball players to improve their shooting skills. He had each player take 20 shots. He recorded how many shots they made individually in the table below.

4	5	5	7	8	8	9	9	9	10	11	16

1. Find the five-number-summary and use it to construct a box plot of the data.

(________ , ________ , ________ , ________ , ________)

Min, Q_1, $Q_2 = M$, Q_3, *Max*

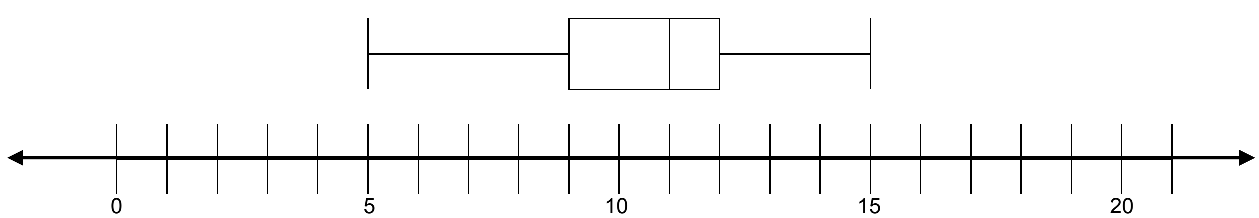


2. Construct a histogram of the data.

Frequency					
	1-5	6-10	11-15	16-20	21-25

Shots Made

After 2 weeks of shooting drills, Coach Watson conducted the same trial again and organized the new data in the box plot below.



3. Compare this data display with the one you created in problem 1. Did the players improve? Use at least three measures of center or spread to support your argument.

SKILL BUILDER 5

1. Multiply $2\frac{2}{3} \times 1\frac{1}{4}$ using an area model. Check using the multiply-across rule.

Multiply using any method. Use the big 1 strategy if appropriate.

2. $\frac{2}{5} \times \frac{5}{8}$	3. $\frac{4}{5} \bullet \frac{1}{2}$	4. $5\left(\frac{2}{7}\right)$
5. $12 \times \frac{5}{8}$	6. $4 \bullet 6\frac{2}{3}$	7. $\left(1\frac{2}{3}\right)\left(2\frac{3}{4}\right)$

8. Wayne wants to make a small pen for his pig. The dimensions of the rectangular pen are $12\frac{1}{2}$ feet by $6\frac{1}{4}$ feet.
   a. How many feet of fencing does Wayne need for the pen?

   b. What will be the area of the pen when it's completed?

SKILL BUILDER 6

1. Alexandria has a board that is 12 feet long. She wants to cut pieces that are each $\frac{2}{3}$ of a foot to make square boxes. How many pieces can she cut?

   a. Draw a diagram to illustrate the problem.

   i. Fill in numbers to rephrase the problem as a fair-share division problem. How can Alexandria divide

   ____ into _____ equal groups?

   b. Write a division problem that illustrates this relationship.

   c. How many pieces can she cut?

   d. Is there any leftover board? If so, how much?

2. Mr. Van has $8\frac{1}{3}$ yards of rope that he needs to cut into pieces that are $\frac{2}{3}$ of a yard for his art students. How many pieces can he cut?

   a. Draw a diagram to illustrate the problem.

   j. Fill in numbers to rephrase the problem as a fair-share division problem. How can Mr. Van divide

   ____ into _____ equal groups?

   b. Write a division problem that illustrates this relationship.

   c. How many pieces can he cut?

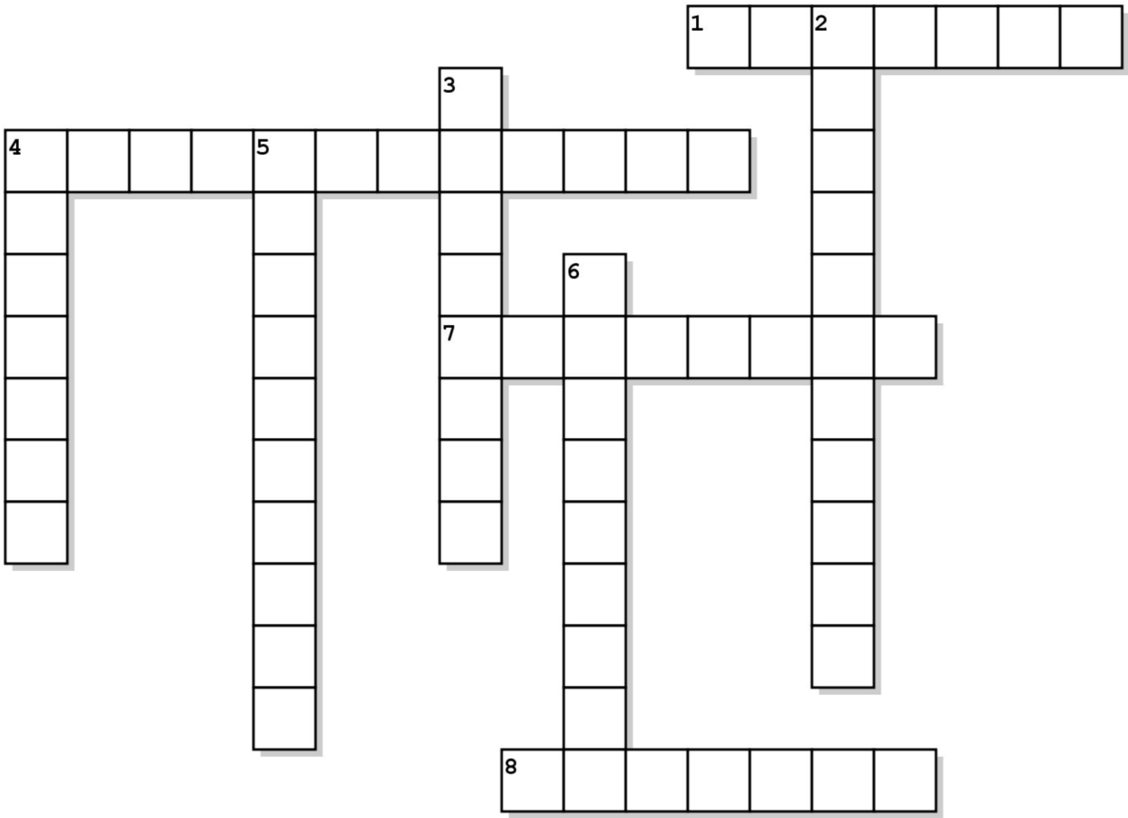
   d. Is there any rope left over? If so, how much?

SKILL BUILDER 7

Compute using any method.

1. $\frac{2}{3} + \frac{1}{2}$	2. $4\frac{4}{5} + 1\frac{1}{3}$	3. $1\frac{4}{7} + \frac{1}{8}$
4. $\frac{2}{3} - \frac{1}{2}$	5. $4\frac{4}{5} - 1\frac{1}{3}$	6. $1\frac{4}{7} - \frac{1}{8}$
7. $\frac{2}{3} \times \frac{1}{2}$	8. $4\frac{4}{5} \bullet 1\frac{1}{3}$	9. $\left(1\frac{4}{7}\right)\left(\frac{1}{8}\right)$
10. $\frac{2}{3} \div \frac{1}{2}$	11. $4\frac{4}{5} \div 1\frac{1}{3}$	12. $1\frac{4}{7} \div \frac{1}{8}$

FOCUS ON VOCABULARY



Across

1 $\frac{2}{3}$ and 4 in the problem $\frac{2}{3} \times 4 = \frac{8}{3}$

4 The property used below

$2 \bullet \left(3\frac{1}{2}\right) = 2 \bullet \left(3 + \frac{1}{2}\right) = 2 \bullet 3 + 2 \bullet \frac{1}{2}$

7 Another name for "the big one"

8 $\frac{8}{3}$ in the problem $\frac{2}{3} \times 4 = \frac{8}{3}$

Down

2 A property: $\frac{1}{2}$ group of 4 gives the same result as 4 groups of $\frac{1}{2}$

3 The result of division

4 Number of groups in a fair-share division problem

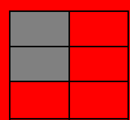
5 Relationship between $\frac{1}{3}$ and 3

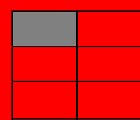
6 Left over in division

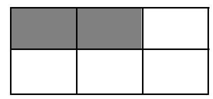
SELECTED RESPONSE

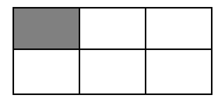
Show your work on a separate sheet of paper.

1. Fabian was trying to create an area model to compute $\frac{2}{3} \times \frac{1}{2}$. Which of the area models below could he use to answer the question? Choose all that apply.

A. 

B. 

C. 

D. 

2. Compute. $\left(5\frac{1}{3}\right)\left(\frac{1}{4}\right)$

A. $5\frac{1}{12}$ B. $1\frac{1}{3}$ C. $5\frac{7}{12}$ D. None of these.

3. This sheet of paper is $8\frac{1}{2}$ inches by 11 inches. How many square inches is the area of this paper?

A. $93\frac{1}{2}$ in^2 B. $19\frac{1}{2}$ in^2 C. 14 in^2 D. $88\frac{1}{2}$ in^2

4. Compute. $1\frac{1}{5} \div \frac{3}{10}$

A. $\frac{18}{50}$ B. 4 C. $1\frac{2}{3}$ D. None of these.

5. Myra needs to cut 100 feet of ribbon into pieces that are $1\frac{1}{4}$ feet long. How many pieces can she cut?

A. 125 B. $101\frac{1}{4}$ C. $98\frac{3}{4}$ D. 80

KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

7.1 Fraction Multiplication

1. Use an area model to illustrate $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$.

2. Compute. $\left(1\frac{1}{4}\right)\left(1\frac{1}{6}\right)$

3. Create a story problem for the expression $15 \bullet \frac{2}{3}$.

7.2 Fraction Division 1

4. Five hikers want to share their remaining $3\frac{1}{3}$ bottles of water equally. How much should each hiker get? Use a division rule or diagrams to justify your answer.

7.3 Fraction Division 2

5. Compute using any method. $2\frac{2}{3} \div 4\frac{1}{2}$

HOME SCHOOL CONNECTION

Here are some questions to review with your young mathematician.

Compute using any method.

1. $\frac{5}{6} \times \frac{3}{8}$

2. $\left(1\frac{1}{3}\right)\left(2\frac{1}{7}\right)$

3. Jessica thinks that dividing by 2 is the same thing as multiplying by $\frac{1}{2}$. Critique Jessica's conjecture and use examples or counterexamples to support your claim.

4. Compute. $5\frac{1}{2} \div 2\frac{4}{9}$

Parent (or Guardian) Signature ______________________________

COMMON CORE STATE STANDARDS – MATHEMATICS

STANDARDS FOR MATHEMATICAL CONTENT	
5.NF.4a*	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction: Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)
5.NF.4b*	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction: Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.NF.6*	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
5.NF.7a*	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions: Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
5.NF.7b*	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions: Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.
5.NF.7c*	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions: Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?
6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

*Review of content essential for success in 6^{th} grade.

STANDARDS FOR MATHEMATICAL PRACTICE	
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP7	Look for and make use of structure.
MP8	Look for and express regularity in repeated reasoning.

9 7 8 1 6 1 4 4 5 0 1 2 2